RESTRICTED CIRCULATION

This paper is not an official document, it is for information only and must not be quoted in any printed publication.

AGRICULTURAL MEMORANDUM NO 824

L D J HARVEY

WIND AND WINDMILLS

Recent events have led to a general awareness of the need to search for sources of energy independent of fossil fuels, resulting in a revival of interest in windmills. The object of this memorandum is to help with the understanding of meteorological implications when considering the use of windmills, with particular reference to the Eastern Region of the Ministry of Agriculture, Fisheries and Food.

Wind is moving air and as such possesses kinetic energy. A measure of that energy is given by

$$K.E. = \tfrac{1}{2} m v^2$$

Where m is the mass of the air and v the velocity.

The mass of air moving horizontally and passing through a vertical cross section of area A in time t is given by

$$m = \rho A v t$$

Where ρ is the density of the air,

$$\text{hence, } K.E. = \tfrac{1}{2} \rho A v^3 t \text{ joules}$$

Thus the maximum power (P max.) available from air moving through a given vertical cross section is given by,

$$P_{max} = \tfrac{1}{2} \rho A v^3 \text{ watts.}$$

(Potential power output in kWm^{-2} is plotted against wind speed in m sec^{-1} at figure 1)

To relate this potential power to a particular area - in this case "The Eastern Region of MAFF" some knowledge of the frequency of various windspeeds and the distribution throughout the area is necessary.

The map at figure 2, showing isopleths of the 75th of the percentile speed, gives a general indication of the areal distribution of wind over the region. Particular attention is drawn to the legend of figure 2 pointing out that the data is valid only for open level terrain between 0 and 70 metres above sea level. Naturally a site on the top of a hill will be more windy than one at the bottom of the valley. The map, which is part of one of a series, was extracted from Meteorological Office Climatological Memorandum No 79 (ref. 1). The complete series of maps of the UK show isopleths of the 25th, 50th, 75th, 90th, 99th and 99.9th percentile speed, which makes the memorandum of particular interest when contemplating the use of wind power.

The publication Met.0,792 (Ref 2) provides, for a selection of stations throughout the United Kingdom, tables of average frequency of wind by speed and direction for each month of the year. From the tables data for two stations, Mildenhall and Shoeburyness, were extracted. Mildenhall is generally representative of areas of low windspeeds and Shoeburyness of those with somewhat higher speeds and thus in conjunction represent the range occurrency across the region. Ogives of cumulative frequency of windspeed, are shown at figure 1, and these together with the power output curve enable estimates of the range of average maximum power available over the region to be made.

For example; in general, the range over the region is of the order of <0.03 to $<0.08 kNm^{-2}$ for 50% of the year (see pecked lines figure 1).

The winds of Mildenhall and Shoeburyness were monitored at heights of 18.3 metres and 27.7 metres above ground respectively, at comparatively open sites. Windspeed near to the surface in general increases logarithmically with height and so to enable the ogives at figures 2 and 5 to be related to other heights a nomogram is provided (figure 3).

Air movement near to the surface is influenced by surface roughness and consequently the windspeed at a rural open site (such as the Mildenhall and Shoeburyness anemometer sites) would usually be less than that at a comparable urban site.

To show how the availability of wind energy varies throughout an average year the maximum output of energy in KWhr per square metre of vertical cross section of air (or square metre of windmill rotor) for each month at both sites was calculated and a histogram drawn (figure 4). These calculations are based on the average frequencies of windspeed extracted from Met 0,792. For each month the long term average frequencies of windspeed of 0, 1-3 knots, 4-6kt, 7-10kt, and thereafter of 5kt class intervals are given. For simplicity in calculating $\frac{1}{2}\rho v^3$ for each class the middle of the range value of v was used, ie $(\overline{V})^3$ and not $\overline{v^3}$. This can be justified by arguing that as the distribution of observations is positively skewed the more significant higher windspeed classes above the mode have fewer occasions towards the upper end of the class and therefore there would be a positive bias in the use of $\overline{v^3}$. Also the class intervals are relatively small and as these decrease $(\overline{V})^3$ approaches $\overline{v^3}$. Energy is expressed in kWhr (figure 4) because of the everyday use of these units.

It is interesting to note the large differences in potentially available energy between the two sites. Also of interest is that peak energy is available at a time when demand is usually high (ie in the winter).

Kinetic energy available to a windmill is that given up by the air meeting the rotor or sails (NB Not that possessed by the air passing through the total area the rotors sweep out.)

The potential power available per square metre of rotor has an average value of 890 kWhrs per year at Mildenhall (windspeeds measured at 18.3m). At Shoeburyness the figure is 2190 kWhrs/m^2/year. (Windspeeds measured at 27.7m). Unfortunately no machine is 100% efficient, so in practice the energy available for transmission will be considerably lower. For a wind powered electrical generator an efficiency between 25% and 35% is commonly quoted. Using such a machine the available energy per square metre of rotor now becomes, on average,

Mildenhall	220 to 310 kWhr yr^{-1}
Shoeburyness	550 to 770 kWhr yr^{-1}

Modern windmills by design (eg by 'feathering' the blades) have upper limiting speeds above which the output remains constant. On certain older machines the rotor is locked parallel to the wind direction at a critical speed thereby stopping generation of electricity completely.

Pontin (ref 3) quotes optional upper limiting speeds (V_R) of 8m sec^{-1}, 10m sec^{-1}, 13.4m sec^{-1} and 15m sec^{-1} for his windmills.

A windmill, however well designed, must have a moment of inertia however small and friction. This means that the wind speed at which the rotors or sails begin to rotate and also the speed at which they stop rotating will be greater than zero, the amount depending on design. As the stopping speed is lower than the starting speed, on some machines by using reserve power (eg batteries), it is possible to use the electrical generator as an electric motor to start the blades revolving when the windspeed lies between the starting and stopping speeds. Even so the estimated average annual available energy will be reduced. An estimate of the loss can be obtained from figure 5, providing the stopping and starting speeds are known.

Finally to give some idea of the output of modern windmills, curves of estimated power output of various sizes of windmills with windspeed have been drawn (figure 5).

As windspeed increases with height (figure 3) the windspeed at the tip of a rotor at its zenith will be greater than that at its nadir for one rotating in the vertical plane. Windspeed at figure 5 is that of the centre of rotation.

Pontin also gives the relationship of the maximum power output (P_R) with V_R (upper windspeed limits) as

$$P_R = .00048\ D^2 V_R^3 Co$$

where D = diameter of rotor and Co = output coefficient of the complete installation (typically 0.25 to 0.35).

For estimation of actual windmill outputs (fig 5) it is assumed that

$$P = .00048\ D^2 V^3 Co$$

Where P = power output for a given windspeed V and Co has an average value 0.30.

Starting and stopping speeds have not been included in this instance as they were not given in Pontin's paper, (estimations are difficult and therefore could be misleading). It must be added this point is of importance when considering installing a windmill since high stopping and starting speeds would result in the rotor being idle for an even more significant part of the time, (ie greater than the frequency of zero wind).

In conclusion it is hoped that this memorandum will be of assistance to those considering use of wind power. The principles used in determining the potential power output of windmills are not specific to a particular area, although the examples themselves use data from sites in East Anglia, and so should not be applied elsewhere.

This memorandum is not intended to offer any recommendations for or against the use of windmills but rather to provide meteorological information supplementary to that of the manufacturer.

Any comments or suggestions about this paper should be addressed to the author.

December 1977
Meteorological Office
Cambridge.

REFERENCES

(1) Caton, P G F — 1976 — 'Maps of Hourly Mean Wind Speed over the United Kingdom 1965-73' Meteorological Office Climatological Memorandum 79, HMSO.

(2) Meteorological Office — 1968 — Met 0,792 Tables of Surface Wind Speed and Direction over the United Kingdom. London HMSO.

(3) Pontin, G W-W — 1977 — 'Energy on the Farm - Wind Power' Wind Energy Supply Co Ltd.

EASTERN REGION

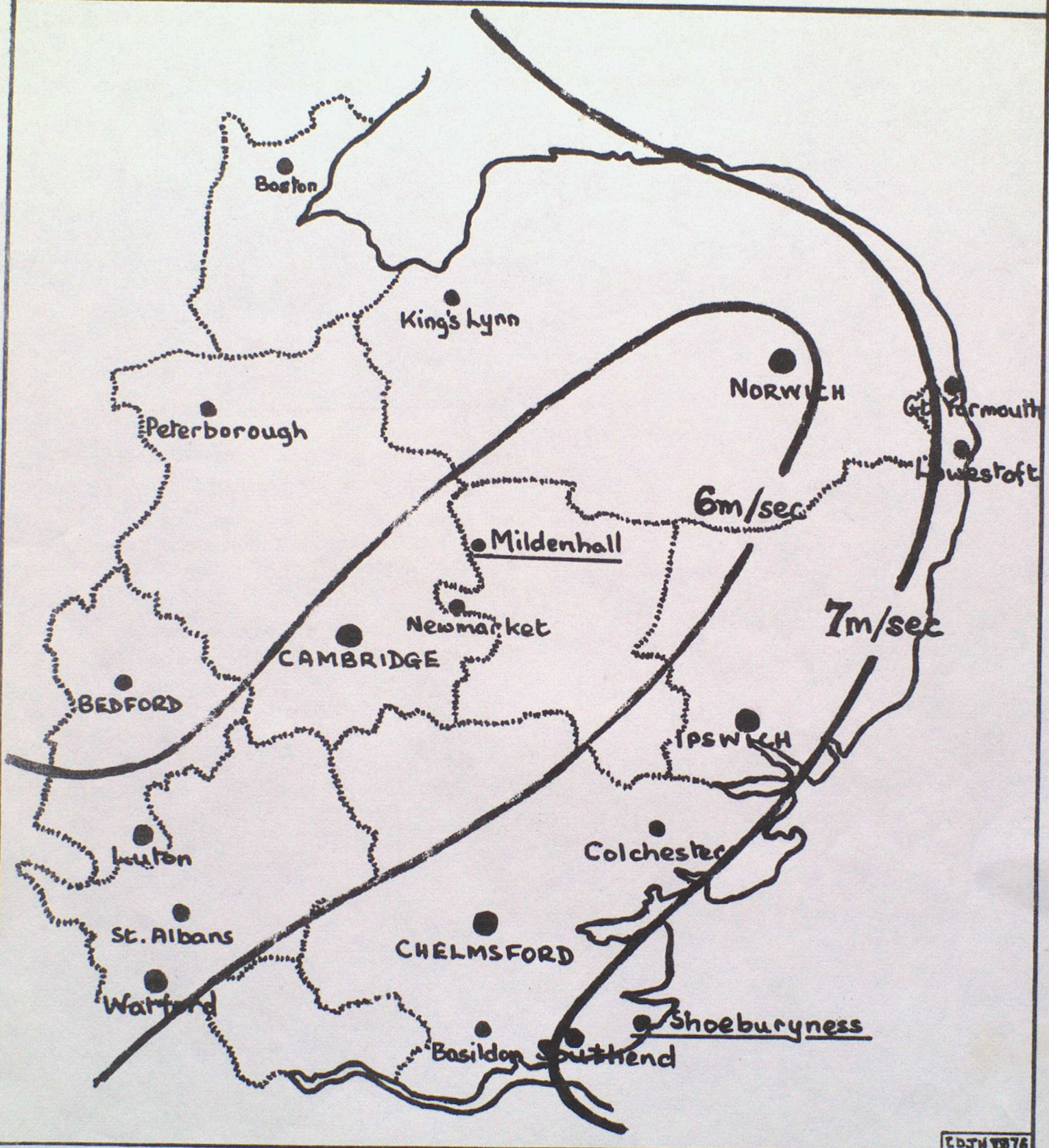


FIG. 1. HOURLY MEAN WINDSPEED IN METRES PER SECOND EXCEEDED FOR 25% OF THE TIME DURING THE YEARS 1965-73. VALID FOR A HEIGHT OF 10m. ABOVE OPEN LEVEL TERRAIN AND FOR ALTITUDES BETWEEN 0 AND 70m. A.S.L.

FIG. 2.
RELATIONSHIP BETWEEN LONG TERM AVERAGE FREQUENCY OF WINDSPEED AND POTENTIAL POWER OUTPUT.

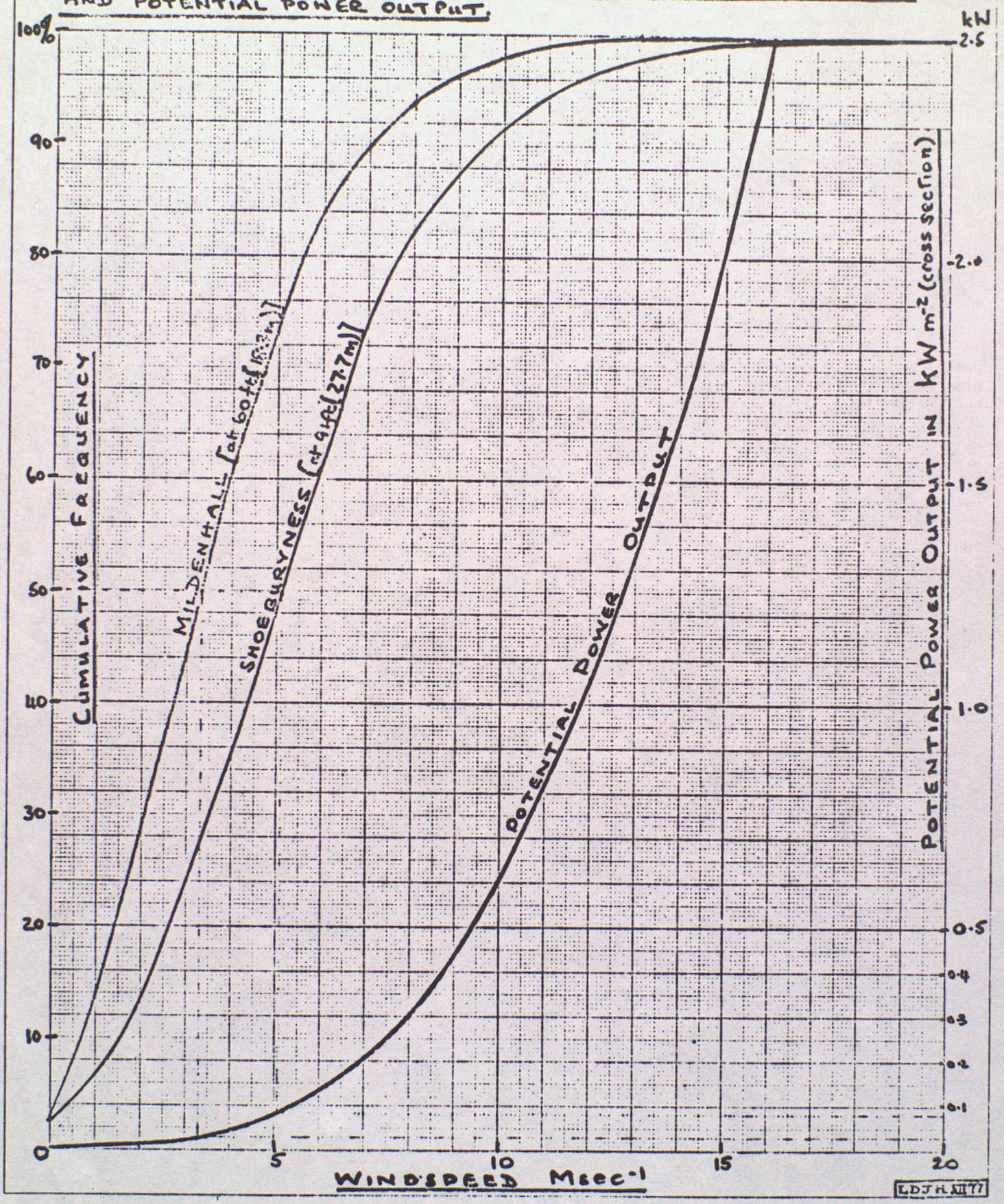


FIG. 3.

NOMOGRAM TO INDICATE WINDSPEED AT ANY HEIGHT (≤200m) GIVEN THE WINDSPEED AT ANY OTHER HEIGHT (≤200m).

(derived from Hellmann's formula)

TO USE:

- Enter at given windspeed on bottom scale and move vertically upwards to line of given height,
- move horizontally to line of height of interest,
- move vertically downwards to windspeed scale and read off.

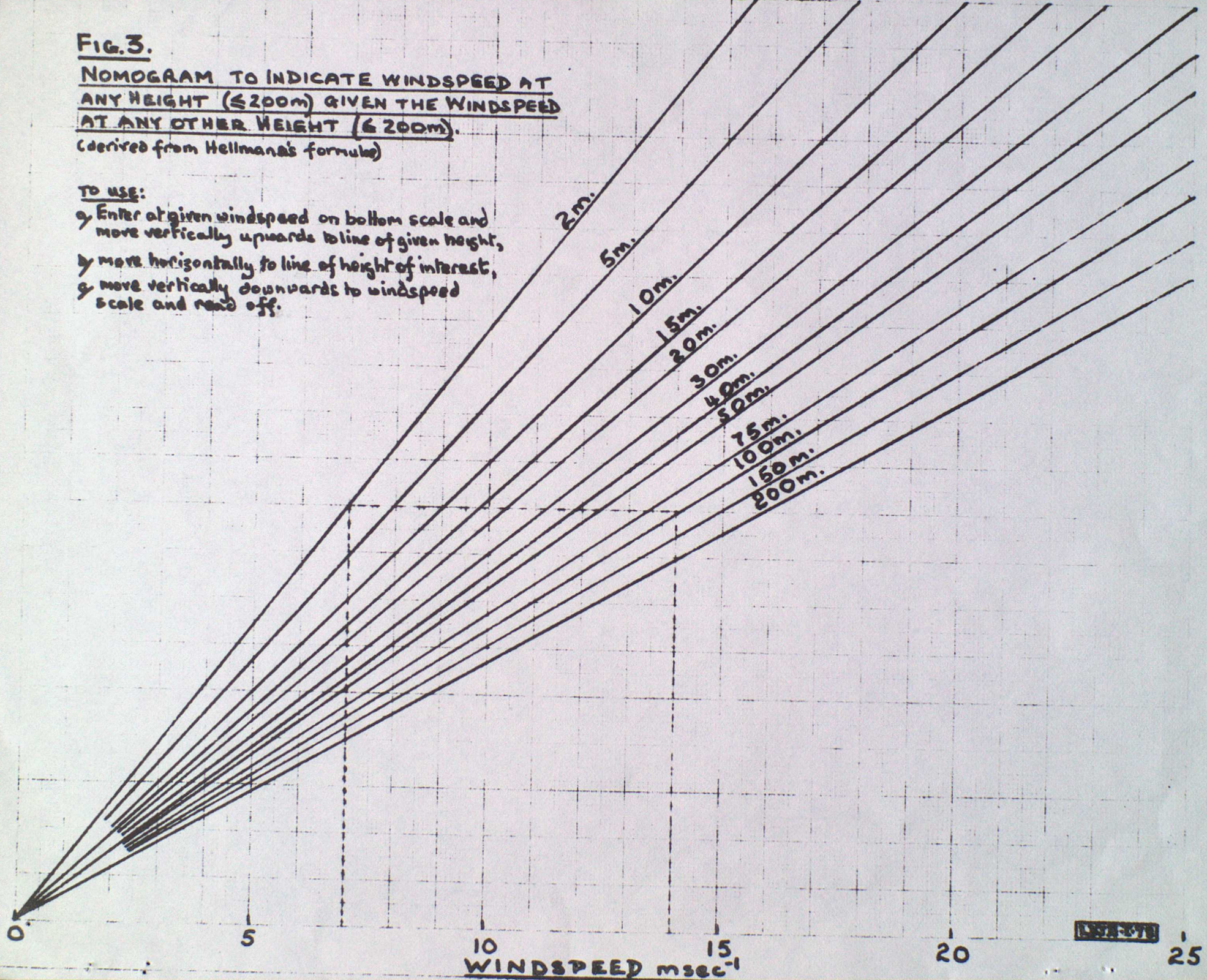
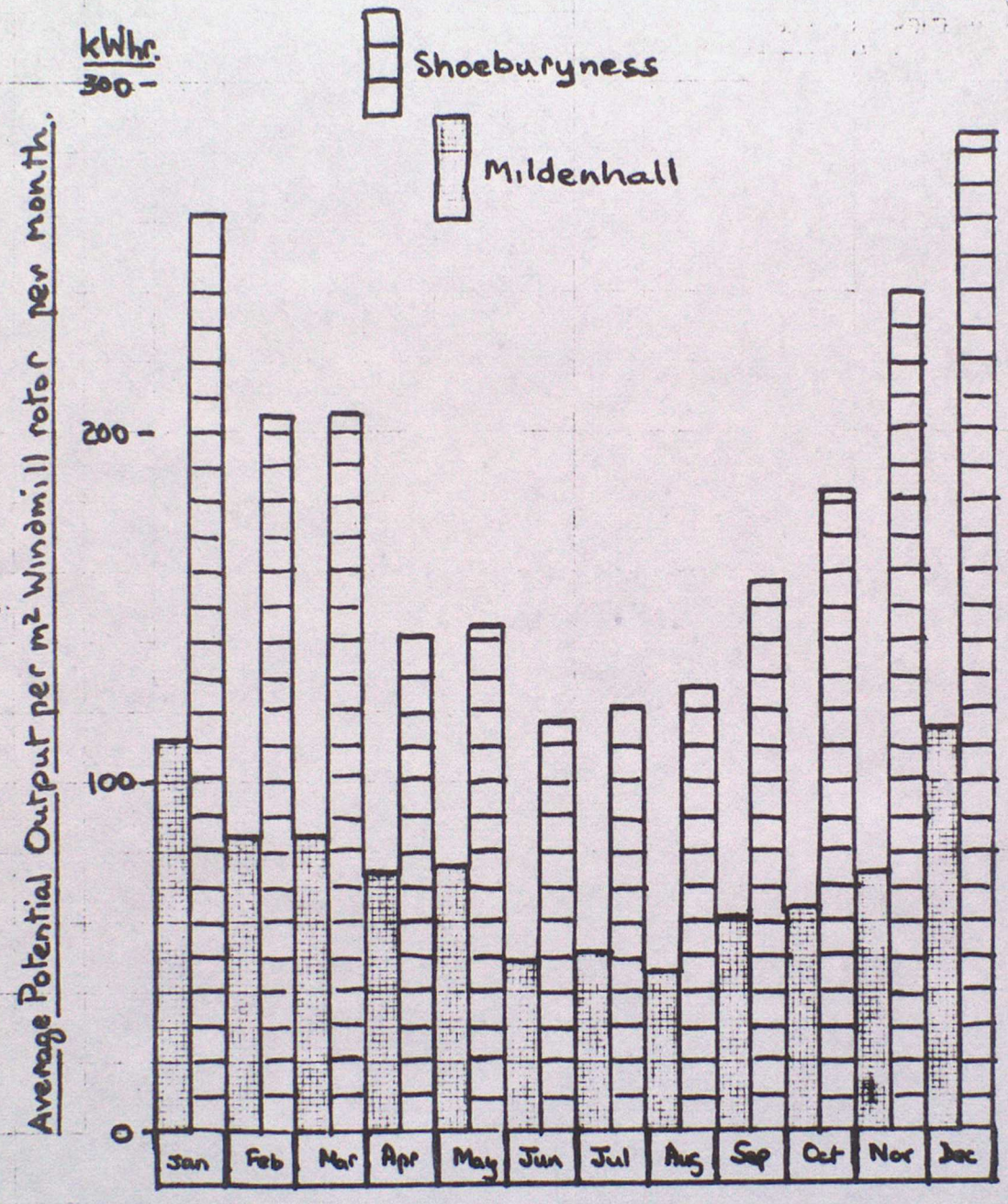


Fig 4



LDTH XII 77

FIG 5

RELATIONSHIP BETWEEN LONG TERM AVERAGE FREQUENCY OF WINDSPEED AND POWER OUTPUT OF WINDMILLS OF VARYING SIZE FOR TWO SITES IN THE E. REGION OF MAFF.